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REMARKS ON η-OPEN SETS

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ABSTRACT

Recently, Subbulakshmi and her research collaborators introduced and studied the notions of η-open sets and η-continuity. The purpose of this paper is to investigate the properties of η-open sets in accordance with semiopen sets. It has been established that the class of η-open sets is nothing but the class of all semiopen sets introduced by Levine in 1963 and the class of gη-closed sets is the class of gs-closed sets.

KEYWORDS: *Topology, Semiopen, Semicontinuity, η-Open, η-Continuity*

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INTRODUCTION

Levine [4] studied the concept of semiopen sets in 1963. He studied some of the properties of topological spaces by replacing open sets by semiopen sets. The complement of a semiopen set is semiclosed. As semiopen sets are very close to open sets, semiopen sets are called nearly open sets in topological spaces. Following this several topologists studied several versions of nearly open sets. In particular Arya and Nour [1] discussed the concept of gs-closed sets. Recently, Subbulakshmi et al. introduced and studied the notions of η-open sets and η-continuity. They also studied the properties of η-interior and η-closure operators. For example gη-closed sets have been examined them. The purpose of this paper is to investigate the properties of η-open sets in accordance with semiopen sets. It has been established that the class of η-open sets is nothing but the class of semiopen sets and the class of gη-closed sets is the class of gs-closed sets.

PRELIMINARIES

Throughout this paper (X,τ) is a topological space and A, B are the subsets of X. The interior and closure operators on A are respectively denoted by *int* A and *cl* A.

DEFINITION 2.1: A is called

- semiopen[4] in (X,τ) if there exists an open set U with $U\subseteq A\subseteq cl\, U$
- η-open [5] in (X,τ) if $A\subseteq Int Cl\, Int A\cup Cl\, Int A$

The complements of the semiopen and η-open sets are known as semiclosed and η-closed sets respectively.

NOTATIONS 2.2:

- scl A = the semiclosure of A.
- sint A = the semi interior of A.

- ηcl A = the semiclosure of A.
- ηint A = the semi interior of A.

LEMMA 2.3: [4] the set A is

- Semiopen if and only if $A \subseteq$ cl intA.
- Semiclosed if and only if int cl $A \subseteq A$.

DEFINITION 2.4: A is called

- Gs-closed [1] if scl $A \subseteq U$ whenever $A \subseteq U$, U is open.
- gη-closed [6] if ηcl $A \subseteq U$ whenever $A \subseteq U$, U is open

The complements of gs-closed and gη-closed sets are called gs-open and gη-open sets respectively.

LEMMA 2.5: [5] the set A is η-closed iff *cl int cl* A $\cap$ *int cl* A $\subseteq$ A.

DEFINITION 2.6: A function $f : (X,\tau) \rightarrow (Y, \sigma)$ is called

- Semicontinuous [4] if $f^{-1}(V)$ is semiopen for each open set V of Y.
- Contra semicontinuous [3] if $f^{-1}(V)$ is semiclosed for each open set V of Y.
- Gs-continuous [2] if $f^{-1}(V)$ is gs-open for each open set V of Y.
- contra gs-continuous [3] if $f^{-1}(V)$ is gs-closed for each open set V of Y.

DEFINITION 2.7: A function $f : (X,\tau) \rightarrow (Y, \sigma)$ is called

- η-Continuous [7] if $f^{-1}(V)$ is η-open for each open set V of Y.
- Contra η-continuous [7] if $f^{-1}(V)$ is η-closed for each open set V of Y.
- gη-continuous [7] if $f^{-1}(V)$ is gη-open for each open set V of Y.
- Contra gη-continuous [7] if $f^{-1}(V)$ is gη-closed for each open set V of Y.

Semiopen versus η-Open

The following lemma is always true for any subset A of a topological space (X,τ).

LEMMA 3.1:

- *int cl int* A $\cap$ *cl int* A = *int cl int* A
- *int cl int* A $\cup$ *cl int* A = *cl int* A
- *cl int cl* A $\cap$ *cl int* A = *cl int* A
- *cl int cl* A $\cup$ *cl int* A = *cl int cl* A
- *int cl int* A $\cap$ *int cl* A = *int cl int* A

- $int\ cl\ int\ A \cup int\ cl\ A = int\ cl\ A$
- $cl\ int\ cl\ A \cap int\ cl\ A = int\ cl\ A$
- $cl\ int\ cl\ A \cup int\ cl\ A = cl\ int\ cl\ A$

PROOF: If A is a subset of (X,τ) then the following two inclusion relational equations hold for the interior and closure operators.

RELATION 3.2: $int\ A \subseteq int\ cl\ int\ A \subseteq int\ cl\ A \subseteq cl\ int\ cl\ A \subseteq cl\ A$

RELATION 3.3: $int\ A \subseteq int\ cl\ int\ A \subseteq cl\ int\ A \subseteq cl\ int\ cl\ A \subseteq cl\ A$

The results follow from Relation 3.2 and Relation 3.3.

PROPOSITION 3.4: If A is a subset of (X,τ) then

- A is η-open if and only if it is semiopen,
- A is η-closed if and only if it is semiclosed,
- $\eta int\ A = s int\ A$
- $\eta cl\ A = s cl\ A$

PROOF:

Suppose A is a subset of X. Then follows from Lemma 3.1, Definition 2.1 and Lemma 2.3; follows from Lemma 2.3, Lemma 2.5 and the results respectively

PROPOSITION 3.5: If A is a subset of (X,τ) then

- A is gη-closed iff it is gs-closed
- A is gη-open iff it is gs-open

PROOF:

Suppose A is a subset of X. Then Definition 2.4 and Proposition 3.4 . Also follows easily by taking complements

PROPOSITION 3.6: A function f: (X,τ)→(Y, σ) is

- η-continuous if and only if it is semicontinuous
- contra η-continuous if and only if it is contra semicontinuous
- gη-continuous if and only if it is gs-continuous
- Contra gη-continuous if and only if it is contra gs-continuous.

PROOF

Let f: (X,τ) →(Y, σ) be a function. Then (i) and (ii) follow from Definition 2.7 and Proposition 3.4. Also (iii) and (iv) follow from Definition 2.7 and Proposition 3.5.

CONCLUSIONS

It has been established that the study of η-open sets, gη-closed sets, η-coninuity, gη-continuity, contra η-coninuity and contra gη-continuity is respectively equivalent to the study of semiopen sets, gs-closed sets, semiconinuity, gs-continuity, contra semiconinuity and contra gs-continuity. Therefore most of the concepts introduced by Subbulakshmi et al. [5, 6, 7] are the existing concepts that are available in the literature of topology.

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